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I: Heat Input and Eigenfunctions

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A THREE DIMENSIONAL MODEL OF THERMOSPHERIC DYNAMICS:

I: HEAT INPUT AND EIGENFUNCTIONS

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ABSTRACT

A three dimensional model of thermospheric dynamics is developed in terms of the eigenfunctions of the atmospheric system. Formulae for the external heat inputs like solar XUV-radiation and corpuscular heating during geomagnetic storms are derived in terms of these eigenfunctions. Those series contain tidal components (depending on local time) and planetary components (depending on seasonal time) which are the generators of tidal and planetary neutral atmospheric waves. The relative importance of corpuscular heating when compared with XUV-heating is estimated. Approximate analytic solutions for the generation and propagation of the atmospheric waves within the dissipative thermosphere, excited by the solar heat input, are given. It is shown that the eigenfunctions which are the Hough-functions within the nondissipative lower atmosphere change into the spherical surface functions within the dissipative thermosphere. Moreover, the density amplitudes of the wave modes decrease proportional to $1/n^2$ where n is the zonal wave domain number of the spherical harmonics. Therefore, only the wave modes with low wave domain numbers n are significant at thermospheric heights.

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I. INTRODUCTION

The dynamic behavior of the neutral component of the upper atmosphere above about 100 km altitude has been studied extensively in the last decade by means of direct in situ measurements of the neutral density (e.g.: Taeusch et al., 1968; Newton, 1970), the neutral wind (e.g.: Vasseur, 1969) or by satellite drag observations (e.g. Champion, 1970; Jacchia, 1970). These observations must be compared and confronted with variations in the ionized component of the thermosphere as inferred from ionospheric drifts (Harnischmacher and Rawer, 1968; Sprenger and Schmindler, 1967), meteor trail observations (Roper and Elford, 1965; Greenhow and Lovell, 1960), and geomagnetic variations (Matushita, 1967), as well as from measurements of electron density (Rawer and Suchy, 1967) ion composition (von Zahn, 1970) and ion temperature (Mahajan, 1969); Waldteufel and McClure, 1969). The available data of the neutral component of the thermosphere reveal the following general picture of thermospheric dynamics:

(a) The diurnal component strongly dominates the daily density variation at thermospheric heights above 200 km with relative amplitudes up to 0.5. The relative amplitude and phase remain roughly constant with altitude above about 300 km altitude. The semidiurnal component above 200 km is weak though not negligibly small when compared with the diurnal component;

(b) The meridional structure of the diurnal density variation during equinox follows roughly a simple sin θ - law, where θ is the co-latitude;

(c) The annual variation is relatively weak. Below 400 km altitude the density bulge follows the zenith angle (Jacchia and Slowey, 1967). On the other hand, there exists a relatively strong semiannual variation within the whole thermosphere which can be observed down to the stratosphere (e.g.: Cook, 1970; Cole, 1968);

(d) The average neutral density as well as the diurnal density amplitude strongly depend on solar activity indicating the dominant influence of the solar XUV heat input on thermospheric dynamics;

(e) During geomagnetic storms the density increases significantly suggesting additional heating of the thermosphere by solar corpuscular radiation and Joule heating.

Some of these observations are consistent with ionospheric data. E.G., the dynamic behavior of the ionospheric F-layer can partly be interpreted by means of neutral winds deduced from the neutral pressure field (Kohl and King, 1967). The wind field deduced from the geomagnetic Sq current at 115 km altitude shows a predominant diurnal component (Kato, 1956; Stening, 1969; Tarpley, 1970). However, some ionospheric observations are in apparent disagreement with measurements of the neutral component. E.G., meteor trail measurements and

ionospheric drift measurements at higher latitudes reveal the predominance of the semidiurnal wind component up to F-layer heights (Greenhow and Lovell, 1960; Harnischmacher and Rawer, 1968; Sprenger and Schmindler, 1967). The neutral temperature inferred from Thomson-scattering measurements gives maximum values at 16.00 local time which seems to be in disagreement with the density maximum at 14.00 local time as determined from satellite and rocket born measurements (Mahajan, 1968).

The neutral and ionized components of the thermosphere form a coupled dynamic system which reacts to the gravitational tidal forces of sun and moon, to the solar XUV-radiation, to solar corpuscular radiation and indirectly to the total solar radiation via dynamic coupling from the lower atmosphere.

A theory of thermospheric dynamics should be able to interpret the above mentioned observations in a selfconsistent manner and to explain the discrepancies between the observations of the neutral and the ionized component. Such theory should lead to a clear separation between the various energy inputs and their influence on thermospheric dynamics depending on day, season and solar cycle. Because of the complexity of the problem we are far away from a final theory of such kind.

Harris and Priester (1962) were the first to attack this problem. They calculated the thermodynamic reaction of a vertical air column to the solar EUV heat input. Considering heat conduction as the most important energy transport,

process at thermospheric heights they were able to reproduce qualitatively the observed diurnal behavior of the neutral thermospheric density. However, quantitatively their results revealed a discrepancy in amplitude and phase between theory and experiment which they resolved by introducing an unknown second heat source.

Lagos and Mahoney (1967) extended this model to other latitudes and showed that the EUV heat input alone is not sufficient to create the observed large thermospheric temperatures at high latitudes. They concluded that horizontal energy transport or an other heating mechanism must be important for the heat balance of the thermosphere. Dickinson et al. (1968) calculated a two dimensional model along a zonal strip. They included horizontal longitudinal winds and showed that in their model the time of the diurnal density maximum shifted toward the early afternoon as the result of adiabatic heating due to vertical motions.

A two dimensional equatorial model has been calculated by Volland and Mayr (1970). Here the radiation condition of characteristic waves within the thermosphere has been applied as boundary condition. This allows the separation of the contribution from the various heat sources within and outside the thermosphere. It has been shown that within the lower thermosphere a tidal diurnal wave generated below 120 km height dominates the dynamic features while above 200 km the thermospheric dynamics are excited primarily by the EUV-heat input within the thermosphere. The second heat source of Harris and Priester could be eliminated by the introduction of horizontal longitudinal winds. These winds are restrained by collisions with the ionospheric plasma. Heat advection due to these

winds shifts the phase of the density amplitude from 17 hours LT to between 14 and 15 hours LT (depending on the numerical value of the collison frequency) consistent with the observations, and it reduces the density amplitude to the observed values.

The two dimensional model of Volland and Mayr (1970) has been adopted to reproduce quantitatively the dependence of the density on solar activity (Volland, 1969a). Likewise, amplitude and phase of the geomagnetic activity effect, of the 27-day rotation effect, and the semiannual effect have been calculated and show satisfactory agreement with the observations.

A first approach to the next step - a threedimensional thermospheric model - has been made by Lindzen and Blake (Lindzen, 1970; Lindzen and Blake, 1970). They used the concept of equivalent plane gravity waves to study the generation and propagation of some of the more important diurnal and semidiurnal tides within the thermosphere.

The model of Lindzen and Blake includes the assumption that the latitudinal structure and the equivalent depth of the various tidal modes do not change with height within a dissipative atmosphere. However, it is well known (e.g. Siebert, 1961) that the eigenfunctions of atmospheric dynamics on the rotating earth which are the Hough functions within the lower atmosphere, become the spherical functions if one can neglect the Coriolis force. The neglect of the Coriolis force is indeed a reasonable approximation at F-layer heights and above where ion-neutral drag and the viscosity forces by far exceed the Coriolis force.

In this paper which consists of three parts we shall study various tidal and planetary wave modes, their latitudinal dependence, height structure and their generation and propagation within thermospheric altitudes. In this first part we shall outline an analytical approach to describe these various modes with the aim of providing insight into some of their general characteristics. For that purpose a number of approximations are necessary.

The main approximations adopted are the use of perturbation theory and the restriction to gravity waves. It is well known that at thermospheric heights two other wave types — heat conduction waves and viscosity waves — can exist. They influence mainly the amplitudes of temperature and horizontal winds. However, at least the influence of heat conduction waves on thermospheric dynamics has already been studied in a two dimensional model (Volland and Mayr, 1970) and this indicates that in our simplified model the temperature amplitude is subject to errors which are not too serious and which should be tolerated considering other uncertainties involved and considering the simplifications gained.

In this part, we shall furthermore develop the solar XUV and corpuscular heat inputs as well as Jacchia's (1964) exospheric temperature distribution into series of spherical harmonics. This will show that only few components in this series with low wave domain numbers are required for a sufficiently accurate representation of the observed temperature distribution. It suggests that the spherical functions approximate rather well the eigenfunctions of the thermospheric dynamic system. Finally, we shall determine in this paper analytic solutions of the generation

and propagation of the various eigenfunctions within an isothermal lower atmosphere and within an isothermal thermosphere.

The analytic solutions developed in this first part shall be used in the second and third part of this paper to calculate explicitly the wave propagation of several important tidal and planetary wave modes which are generated by the corresponding components of the solar heat input. These waves include the fundamental symmetric diurnal and semi-diurnal modes, the antisymmetric diurnal mode and the annual and semiannual modes. In the second and third part of this paper we shall study moreover in detail the altitude variations in the latitudinal structure for these modes. Especially, it will be shown how the eigenfunctions which are in the Hough functions within the lower atmosphere transfer into the spherical functions at thermospheric heights. Furthermore a detailed analysis of the wind systems of the various wave modes will be given.

A more sophisticated numerical study of the three dimensional thermospheric dynamics including heat conduction waves and taking into account realistic temperature profiles within the thermosphere will be given in an additional paper (Volland and Mayr, 1971a (referred to as paper I)).

2. THE MODEL AND ITS RESTRICTIONS

a. Application of perturbation theory

In order to find tractable analytic solutions for the three dimensional spherical model of thermospheric dynamics we have to restrict outselves to a model which

is as simple as possible. The most important approximation in our approach is the use of perturbation theory. That allows us to treat the various eigenfunctions of the atmosphere as if they were decoupled from each other and to separate the thermospheric disturbances related to the various energy resources of different time scale. The usual method of numerical analysis of thermospheric data in fact implicitly adopts the same assumption when one resolves the various perturbation periods like diurnal, semidiurnal, annual periods ect. by means of some kind of spectral analysis. If coupling between the individual eigenfunctions would be significant, one had to expect a significant contribution from higher harmonics of the basic periods as well as mixed frequencies. This has not been observed.

We can estimate the errors involved in such a perturbation treatment. The horizontal and vertical winds at thermospheric heights are of the order of

$$|u| \lesssim 100 \text{ m/sec}$$

$$|w| \lesssim 1 \text{ m/sec}$$

(Volland and Mayr, 1970). The total time derivative in the equations of momentum and of energy conservation is for the diurnal tidal waves

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{v} \cdot \text{grad}) \sim j\,\Omega + j\,k_x\,u + j\,k_z\,w$$

with

$k_x \sim 1.5 \times 10^{-4}\ \mathrm{km}^{-1}$ (horizontal wave number)

$k_z \sim \frac{1}{2H_0} \sim 10^{-2}\ \mathrm{km}^{-1}$ (vertical wave number)

$\Omega = 7.27 \times 10^{-5}\ \mathrm{sec}^{-1}$ (angular frequency of the earth's rotation)

H_0 is the pressure scale height. Thus, the nonlinear terms are of the order

$$\frac{k_x\,|u|}{\Omega} \sim \frac{|k_z\,w|}{\Omega} \lesssim 0.2.$$

They still allow a perturbation treatment if one accepts errors of the order of 20%. If one considers the uncertainties in the experimental data as well as in our knowledge of the energy sources and the physical coefficients like heat conductivity, viscosity and ion-neutral collisions at thermospheric heights, the error due to the perturbation approximation seems tolerable. Moreover, it will be shown in the following sections that the thermosphere behaves like a low pass filter for tidal and planetary waves. Thus it suppresses waves with great wave domain numbers and therefore prevents the generation of higher harmonies due to nonlinear coupling.

For the perturbated thermosphere the barometric height formula is assumed to be valid. That implies the neglect of the vertical inertial force which, when compared with the earth's acceleration force g leads to the inequality

$$|w| \ll \frac{g}{\Omega} \sim 10^5\ \mathrm{m/sec}$$

valid for all tidal and planetary waves. Moreover, we shall neglect the horizontal viscosity forces and the horizontal heat fluxes which are small compared with the equivalent horizontal and vertical components respectively below 400 km:

$$\left|\frac{\eta\, \partial^2 u/\partial x^2}{\eta\, \partial^2 u/\partial z^2}\right| \sim \left|\frac{K\, \partial^2 T/\partial x^2}{K\, \partial^2 T/\partial z^2}\right| \sim \left|\frac{k_x^2}{k_z^2}\right| \sim \frac{4\, H_0}{R_0^2} \cdot 2 \times 10^{-4}$$

(u = horizontal wind; T = temperature; η = coefficient of viscosity; K = coefficient of heat conduction; R_0 is the earth's radius). Finally, we shall neglect the metric factors in the equations of mass and energy due to the spherical coordinates (r is the distance from the earth's center and $z = r - R_0$):

$$\frac{1}{r^2}\,\frac{\partial}{\partial r}\,(r^2 \ldots) \sim \frac{\partial}{\partial r} = \frac{\partial}{\partial z}$$

because of

$$\frac{2}{k_z\, r} \simeq \frac{4\, H_0}{R_0} \sim 3 \times 10^{-2}$$

Perturbation theory presupposes the knowledge of the mean physical parameters of the steady state atmosphere. We shall adopt as such steady state atmosphere the Jacchia model (Jacchia, 1964) and consider the vertical structure of this model as the quiet thermosphere during a particular solar activity. All variations in time and space due to temporal and spatial variations of the heat sources are considered as perturbations superimposed upon that quiet thermosphere.

We shall also neglect the temporal variations in the neutral composition. In the case of tidal waves such neglect leads to an underestimation of the temperature amplitude which however is not too serious (Volland, 1969a). In the case of some types of planetary waves the wind induced variations in the neutral composition may give rise to substantial errors at heights where atomaric oxygen predominates. This aspect of the thermosphere dynamics will be investigated in separate studies.

b. Restriction to gravity waves

Within the lower atmosphere, tidal waves are of the type of internal gravity waves. Within the thermosphere, dissipation effects due to heat conduction and molecular viscosity give rise to the generation of three new wave modes: heat conduction waves, and two kinds of viscosity waves (Volland, 1969b). These wave types become increasingly important with increasing height and contribute especially to the temperature and horizontal wind amplitudes of the tidal variations. In this approach here we shall neglect those waves and restrict ourselves only to the gravity waves. However we shall introduce the effects of heat conduction and viscosity in the propagation of gravity waves by replacing the second derivatives with height of temperature and horizontal winds by height dependent coefficients in the equations of horizontal momentum and energy conservation. That procedure in fact is equivalent to an outfiltering of the three additional wave types.

In paper I we shall take into account the heat conduction waves in the numerical calculations. There, we shall discuss the implications due to the neglect of the additional wave types.

It will turn out that our approach in fact gives results which become only modified by a more exact treatment. Temperature and horizontal wind amplitudes are most affected by our approach. However it is possible to simulate the influence of heat conduction waves and viscosity waves by the introduction of a slightly modified vertical profile of the heat input and by an effective ion-neutral collision number respectively.

The dissipation coefficients are determined in the following manner: From the work of Jacchia (1964) we know that at a given time the vertical temperature profile at thermospheric heights above 120 km can be rather well approximated by the formula

$$T = T_\infty - (T_\infty - T_{120})\, e^{-(z-z_{120})/H} \tag{1}$$

Here T_∞ is the exospheric temperature, T_{120} is the temperature at $z_{120} = 120$ km altitude, and H is a scale factor which is of the order of

$$H = 40 \text{ km}$$

during moderate solar activity. Assuming vertical profiles of the wave amplitudes in the form of equation (1), we obtain for the heat conduction term in the energy equation

$$-\frac{\partial}{\partial z}\left(K\,\frac{\partial T}{\partial z}\right) \sim 2\, Z_{hc}\, \frac{\gamma\, p_0\, \Omega\, T}{(\gamma - 1)\, T_0} \tag{2}$$

with

$$Z_{hc} = \frac{2.5\,\eta\,(T_\infty - T_{120})\,(3\,T - T_\infty)\,e^{-(z-z_{120})/H}}{4\,\gamma\,\Omega\,T^2\,\rho_0\,H^2}$$

Here is ρ_0 = the mean density; T_0 the mean temperature, p_0 the mean pressure, and $\gamma = c_p/c_v \sim 1.5$ the ratio between the specific heats, $\eta = A\sqrt{T}$ the coefficient of molecular viscosity; $K = 2.5\,c_v\,\eta$ the coefficient of molecular heat conductivity (Chapman and Cowling, 1952). It will be shown later that the wind profiles have approximately the same vertical structure as that expressed in (1). Therefore, we find for the viscosity force in the equation of horizontal momentum.

$$-\frac{\partial}{\partial z}\left(\eta\,\frac{\partial u}{\partial z}\right) \sim 2\,Z_{vis}\,\Omega\,\rho_0\,u \qquad (3)$$

with

$$Z_{vis} = \frac{\eta\,(u_\infty - u_{120})\,(3\,u - u_\infty)\,e^{-(z-z_{120})/\bar{H}}}{4\,\Omega\,u^2\,\rho_0\,\bar{H}^2}$$

where $\bar{H}$ is the scale factor of the wind profile. Provided $\bar{H} \sim H$, both dissipation factors Z_{hc} and Z_{vis} are of the same order of magnitude. They tend to become constant with height because of

$$T \to T_\infty,\; u \to u_\infty \text{ and } \rho_0 \propto \exp(-z/H).$$

An estimate using data from the Jacchia model gives

$$Z_{hc} \sim Z_{vis} \begin{cases} \lesssim 0.05 & \text{at } z \lesssim 150 \text{ km} \\ \gtrsim 0.1 & \text{at } z \gtrsim 200 \text{ km} \end{cases}$$

From the more exact numerical study it is inferred that the dissipation factors Z_{hc} and Z_{vis} are indeed of the order of 0.1 to 0.5 above 200 km altitude.

We conclude that molecular viscosity and heat conduction can be neglected when compared with the Coriolis parameter 1/2 below 150 km. However, these terms can play a significant role in thermospheric dynamics at heights above 200 km. In this connection it is worth mentioning that the ion-neutral drag term

$$Z_{col} = \frac{\nu}{2\,\Omega} \tag{4}$$

(ν is the collision number of ion-neutral collisions) is of the order of three at F2-layer heights (Dalgarno, 1964). Therefore, in this height range and for tidal and planetary waves, ion drag is generally more important than molecular viscosity (Geisler, 1967; Kohl and King, 1967).

3. THE DISTRIBUTION OF THE ENERGY INPUT

The thermosphere reacts to solar heating or to gravity forces and behaves like an oscillator system in which forced oscillations are generated. The external energy generator may have any possible spatial and temporal distribution. If our assumption about the validity of perturbation theory is correct, we should be able to develop the energy generator in terms of the eigenfunctions of the thermospheric dynamic system and to study separately the response of the individual thermospheric wave modes to the corresponding energy modes.

It is generally accepted that the influence of the gravitational forces of moon and sun on the atmospheric tides are small when compared with the solar heat input due to solar radiation (Chapman and Lindzen, 1970). Therefore, we shall neglect that excitation mechanism in the following. However, electric fields may be a significant source of wave generation. These fields originating either from the Sq region at E layer heights or from the magnetosphere are mapped up or down along the geomagnetic field lines and give rise to ion drift across the geomagnetic field at F2 layer heights. This ion drift in turn leads to wind and pressure perturbations in the neutral component via ion-neutral drag (Hines, 1965a).

The electric field of the Sq region is generated by the neutral tidal wind at E layer heights. It thus acts like a coupling link between lower and upper thermosphere. It couples a fraction of the kinetic energy of the wind field at E layer heights to the winds at F2 layer heights. Since the wind at E layer heights itself is generated by solar heating within and below that region, such coupling can be considered as a vertical redistribution of the total solar heat input within the atmosphere. In a similar sense the electric fields from the magnetosphere are responsible for the transformation of a fraction of solar corpuscular energy stored within the magnetosphere into wave energy of the neutral thermosphere. They generate electric currents within the ionosphere which in turn heat the thermosphere via Joule heating. They also force the ions to drift across the geomagnetic field lines and induce wind and pressure variations at F layer heights.

The solar heat input into the thermosphere via photo-dissociation, ion recombination, and electron cooling is itself a very complicated process and far from being well understood. A very simple approach which is probably not too far from the reality is to assume that the effective heat input per volume is proportional to the mean pressure at thermospheric heights. This assumption may even include the virtual heat input due to the electric fields. In this paper we adopt that picture with a slight modification. We take a height dependence of the heat input Q like

$$\frac{Q}{P_0} \propto g(z) = e^{(1-\epsilon)(z-z_0)/2H_0} \tag{5}$$

with a constant value of ϵ near, but not necessary equal, to one. $z_0 \sim 100$ km is a reference height above ground. For convenience we shall consider in this paper an isothermal thermosphere, H_0 = const. In the numerical study of paper I we shall use the value $\epsilon = 1$ and a realistic temperature profile of the thermosphere.

It remains to determine the temporal and spatial distribution of the heat source. As basic period we select the time of one year with the angular frequency of

$$\Omega_a = \frac{2\pi}{\text{one year}} = 2 \times 10^{-7}\ \text{sec}^{-1} \tag{6}$$

Higher periods, e.g. the eleven-year-cycle, are slow enough to be treated quasi-stationary.

We first consider the XUV input of the sun. In order to obtain an approximate distribution of the XUV input we assume that its effective heat input is proportional to the zenith angle χ of the sun

$$Q_{XUV} = \frac{(1 + \epsilon)\, \bar{Q}_{XUV}\, g\,(z)}{2\, H_0\, \pi\, r_0^2} \cos \chi \tag{7}$$

for $|\chi| \leq \pi/2$, and zero otherwise. Here $\bar{\bar{Q}}_{XUV}$ is the total height, space and time integrated averaged heat input (in erg/sec) above the reference height $z_0 = r_0 - R_0$ where r_0 is the distance from the earth's center and R_0 is the earth's radius. Though the real distribution may deviate more or less from equation (7) it should nevertheless give the right order of magnitude in particular since we shall deal here primarily with the relative magnitudes of the various frequency components.

We develop the function (7) into a series of spherical functions. Approximating the time of sun rise and sun set by

$$\tau_0 = \mathrm{arc}\ \cos\,(\mathrm{tg}\ \delta\ \mathrm{ctg}\,\theta) \approx \pi/2 - \mathrm{tg}\ \delta\ \mathrm{ctg}\ \theta \tag{8}$$

and taking

$$\cos \delta \sim 1$$

$$\sin \delta \sim \delta$$

where $\delta = \delta_0 \cos \Omega_a t$ is the solar declination with a maximum value of $\delta_0 = 0.4$, t is the universal time ($t = 0$ at the begin of the year) and θ is the polar distance,

we obtain the following series:

$$Q_{XUV} = Q^0_{XUV}\, g\,(z) \Bigg[P^0_0 - 2\,\delta_0 \cos \Omega_a\, t\, P^0_1 - \left\{ \frac{5}{8} - \frac{15}{16}\,\delta^2_0 \cos 2\,\Omega_a\, t \right\} P^0_2$$

$$- \left(2\,P^1_1 - \frac{5\sqrt{3}}{4}\,\delta_0 \cos \Omega_a\, t\, P^1_2 \right) \cos \tau + \frac{5\sqrt{12}}{16}\, P^2_2 \cos 2\,\tau + \ldots\ldots \Bigg] \qquad (9)$$

$$(r \geqq r_0)$$

$Q^0_{XUV} = (1 + \epsilon)\,\bar{Q}_{XUV} / 8\,\pi\, H^2_0\; r^2_0$ (in erg/cm³ sec) is the average heat input per volume at the height z. $P^m_n\,(\theta)$ are the spherical functions in Schmidt's normalization depending on polar distance θ, $\tau = \Omega t + \lambda$ is the local time and λ is the geographic longitude.

Though the development equation (8) breaks down near the poles, the coefficients of equation (9) approximate the energy distribution of (7) with an accuracy of about ±10% which has been tested by an exact numerical calculation. In equation (9) all terms with second or higher powers of δ_0 have been neglected (except those that produce the semiannual component). These neglected terms have magnitudes of the order of 0.1 or smaller.

Turning to the solar corpuscular heating we consider only the mean heat input average over the individual substorms. The heat input due to precipitating fast electrons and electric fields from the magnetosphere is confined to the auroral ovals, and most of the heating occurs on the night time hemisphere where the

component of the polar electrojet flows (Akasofu, 1968). It is well known that the geomagnetic disturbances related to that heat input do not show a significant annual variation. However, the semiannual component with a maximum during the equinox is pronounced (Chapman and Bartels, 1951). The semiannual variation of the u_1 - measure of Bartels can be represented by

$$u_1 \sim \tilde{u} \,(1 - 0.1 \cos 2\,\Omega_a\, t) \tag{10}$$

where $\tilde{u}$ depends on solar activity. u_1 is a measure of the disturbed geomagnetic horizontal component. Thus, the magnetic energy of the disturbed geomagnetic field is proportional to u_1^2 . Assuming proportionality between the magnetic energy and the dissipated heat input, we arrive at a heat source averaged over storm time and space of

$$\bar{Q}_{corp}\,(1 - 0.2 \cos 2\,\Omega_a\, t)\ (\text{in erg/sec}). \tag{11}$$

Assuming that this heat input is deposited within an infinitely thin strip at ±65° latitude on the night time hemisphere, we develop it into spherical harmonics (Volland and Mayr, 1971b).

$$\begin{aligned} Q_{corp} = Q^0_{corp}\, g\,(z)\,(1 - 0.2 \cos 2\Omega_a t)\,\{P^0_0 + 3.66\, P^0_2 + 2.22\, P^0_4 + \cdot\cdot \\ + (0.81\, P^1_1 + 3.58\, P^1_3 + \cdots) \cos \tau + \cdots\} \end{aligned} \tag{12}$$

with

$$Q^0_{corp} = \frac{(1 + \epsilon)\,\bar{Q}_{corp}}{8\,\pi\,H_0\,r_0^2}$$

the average heat input per volume at $z_0 = r - R_0$. Here, we used for convenience the same height dependence for Q_{corp} as for Q_{EUV}. The higher order coefficients in equation (12) may, of course, change significantly if we choose any other latitude and they would decrease if we would use a strip of finite broadness. In any case, these coefficients already indicate that this series converges very slowly, and that we need a great number of terms to approximate sufficiently well our assumed heat input distribution. However, as will be seen in the next section, the atmospheric dynamic system filters out wave modes with great wave domain numbers (n, m) so that in fact only the energy modes of lower degree in equation (12) can excite significant density variations at thermospheric heights.

It should be mentioned here, that this is true only for the storm time averaged corpuscular heat input. Any individual polar substorm may generate short periodic gravity waves which travel from the auroral zones into the lower latitudes (Testud, 1970; Chimonas and Hines, 1970). However, these waves are a local phenomenon and outside the subject of our paper.

A third possible energy input into the thermosphere may have its origin within the lower atmosphere. Such energy input is due to wave energy dissipation of waves generated within the lower atmosphere and penetrating into the dissipative

thermosphere. These waves may include the whole spectrum of short periodic internal gravity waves (Hines, 1965b), the fundamental diurnal tidal wave (Volland, 1969a) or semidiurnal tidal waves (Lindzen and Blake, 1970b). Wave dissipation is a nonlinear process and therefore can not be treated in our perturbation theory. This wave dissipation affects essentially the zero component P_0^0 of the heat input and gives rise to an increase of the mean temperature. We therefore have to introduce an additional heat source

$$Q_{dis} = Q_{dis}^0 (1 - 0.05 \cos 2 \Omega_a t) P_0^0 \tag{13}$$

which contains a semiannual component. The numerical value of this semiannual component is anticipated from the following results. During moderate solar activity it is

$$Q_{dis}^0 \gtrsim Q_{XUV}^0$$

indicating that a significant fraction of the average exospheric temperature is caused by that heat input due to wave dissipation (see Page 3 of this paper).

Before we shall discuss the implications of equations (9), (12) and (13) on thermospheric dynamics, we want to develop also the exospheric temperature distribution T_∞ of the Jacchia model (Jacchia and Slowey, 1967) in terms of spherical harmonics. Jacchia's model is supposed to represent well the density variations above about 250 km. The result is (derived for a solar activity factor of F = 125 and the Jacchia parameters n' = m' = 2.5)

$$T_\infty = T_0^0 \, [P_0^0 - 0.048 \cos \Omega_a t \, P_0^1 - 0.007 \, P_2^0$$

$$+ (0.12 \, P_1^1 - 0.008 \cos \Omega_a t \, P_2^1) \cos (\tau - 15.0^h)$$

$$+ 0.035 \, P_2^2 \cos [2 (\tau - 13.2^h)] - \ldots\ldots] \tag{14}$$

$$T_0^0 \left[- 0.04 \, P_0^0 + \sum_{2n'} c_{2n'} \, P_{2n'}^0 \right] \cos 2 \, \Omega_a t$$

with $T_0^0 = 1000°K$. Here, the first bracket on the right hand side in equation (14) is an excellent approximation of the analytical function of Jacchia's exospheric temperature distribution while the second bracket in equation (14) takes account of the semiannual variation of T_∞ which is eliminated in the Jacchia model. We related the observed semiannual temperature amplitude to the (0.0)-component in equation (14) and indicate our ignorance about any latitudinal dependence by terms with arbitrary coefficients c_{2n}, which should exist due to the corresponding terms in equations (12) and (13) though according to Cook (1970) the semiannual variation does not show a significant latitudinal dependence.

One aim of this paper is to relate the individual terms in equation (14) to the corresponding energy components in equations (9), (12) and (13). That is not possible without some ambiguity because we observe in fact the thermospheric response of the combined energy sources. We shall return to these questions in some detail in the two following parts of this paper. However,

some general conclusions can already be drawn from a comparison of equations (9), (12), (13), and (14).

First, the XUV heat source contains a rather strong zonal (2.0) component which, als already Lagos and Mahoney (1967) pointed out, should give rise to a very low mean exospheric temperature at higher latitudes. The temperature distribution of equation (14), on the contrary, shows a rather weak mean latitudinal variation with the temperature gradient directed towards the poles. Jaccia's first model (with his parameters n' = 2.5 and m' = 1.5) distinguished by an elongated pressure bulge has even a mean temperature gradient directed toward the equator equivalent to a positive (2.0)-component in the series of equation (14). Therefore, this coefficient is probably very small. Such a small mean latitudinal dependence of T_∞ can readily be explained by the auroral heating according to equation (12) which compensates the respective (2,0)-term of the XUV heat input. Thus, we can estimate the ratio between the total heat inputs of corpuscular and XUV heating as

$$\frac{Q_{corp}}{Q_{XUV}} \lesssim \frac{5}{8 \times 3.66} = 0.17 \tag{15}$$

A similar compensation occurs for the diurnal (1.1)-coefficients. However, here the effect is much smaller and yields a reduction of the total (1,1)-component by not more than 7% due to auroral heating.

Secondly, the XUV heat input and the auroral heat input contribute to the (2.0)-component of the semiannual variation in the form

$$\left\{\frac{15}{16}\,\delta_0^2\,Q^0_{XUV} - 0.2 \times 3.66\,Q^0_{corp}\right\} P_2^0 \cos 2\,\Omega_a\, t \gtrsim 0.025\,Q^0_{XUV}\,P_2^0 \cos 2\,\Omega_a\, t$$

if we adopt the estimate of (15). This variation is very small. The observations within thermospheric heights indeed seem to support a rather small latitudinal dependence of the semiannual variation. The main contribution of the heat sources to the semiannual variations therefore appears to come from the (0,0) components of the auroral heat input (equation (12)) and from the heating due to wave dissipation (equation (13)). The sum of the three heat sources is then for the (0,0) component

$$(Q_{XUV} + Q_{corp} + Q_{dis})^0_0 = 1.17\,Q^0_{XUV}\,(1 - 0.029 \cos 2\,\Omega_a\, t)$$
$$+ Q^0_{dis}\,(1 - 0.05 \cos 2\,\Omega_a\, t).$$

With $Q^0_{dis} \gtrsim Q^0_{XUV}$ and $Q^0_{corp} \lesssim 0.17\,Q^0_{XUV}$, we conclude that the predominant generator of the semiannual variation is the energy due to wave dissipation although a small but significant fraction is also generated by XUV and auroral heating.

According to an international convention (Hines, 1970) we shall call tidal waves all waves with periods of one lunar or solar day or a fraction of a day. These waves are obviously generated by those energy modes in equations (9) and (12)

with wave domain numbers $m \geq 1$. Planetary waves are wave modes with periods exceeding one day. They are therefore generated by the energy modes in equations (9), (12) and (13) with a wave domain number $m = 0$. Tidal waves will be discussed in the second part, planetary waves will be treated in the third part of this paper.

4. APPROXIMATE GENERAL SOLUTIONS OF WAVE PROPAGATION

In this section we want to solve simultaneously the equations of conservation of mass, momentum and energy in terms of eigenfunctions of the atmospheric dynamic system taking into account the approximations and assumptions discussed in section 2. In the following parts of this paper we shall use these solutions to discuss several tidal and planetary waves.

According to our assumption in section 2 the quiet steady state structure of the atmosphere is related to the energy input of the (0,0)-components in equations (9), (12) and (13):

$$(Q^0_{XUV} + Q^0_{corp} + Q_{dis})\, P^0_0\, g\,(z)$$

We are interested in any departure in time and space from that quiet atmospheric structure represented by the height-dependent mean values of temperature T_0, density ρ_0, and pressure p_0. The perturbated physical values may be written as temperature T, density ρ, pressure p, northerly wind u, westerly wind v, and

vertical wind w. The equations of conservation of mass, momentum and energy, and the equation of state are then presented for these perturbated amplitudes in spherical co-ordinates (r, θ, λ)

$$\frac{1}{\rho_0}\frac{\partial \rho}{\partial t} + \operatorname{div} \vec{v}_{hor} + \frac{\partial w}{\partial z} - \frac{w}{H_0} = 0$$

$$\frac{\partial u}{\partial t} + 0.6\,\nu u - 2\,\Omega c v - \frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\eta \frac{\partial u}{\partial z}\right) + \frac{1}{\rho_0 r}\frac{\partial p}{\partial \theta} = 0$$

$$\frac{\partial v}{\partial t} + \nu v + 2\,\Omega c u - \frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\eta \frac{\partial v}{\partial z}\right) + \frac{1}{\rho_0 r s}\frac{\partial p}{\partial \lambda} = 0 \qquad (16)$$

$$g\rho + \frac{\partial p}{\partial z} = 0$$

$$c_v \rho_0 \frac{\partial T}{\partial t} + p_0\left(\frac{w}{H_0} - \frac{1}{\rho_0}\frac{\partial \rho}{\partial t}\right) - \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) = Q$$

$$\frac{p}{p_0} = \frac{\rho}{\rho_0} - \frac{T}{T_0}$$

Here, for abbreviation,

$$s = \sin\theta;\; c = \cos\theta;\; \vec{v}_{hor} = (u,\ v).$$

The factor $0.6\ \nu$ in the second equation of (16) takes account of the latitudinal mean of the ion-neutral collision number ν.

The heat source Q in (16) is the sum of solar XUV -, corpuscular heat input and dissipated energy and includes also a heat sink due to infrared cooling of the thermosphere. That cooling may be about 10% in magnitude of the XUV-heat input (Harris and Priester, 1962). But in view of the uncertainties in the adopted heating rate we shall not evaluate this heat sink but rather consider it as included in the effective XUV heat source.

We develop the wave amplitudes of pressure p, temperature T, density ρ, and vertical wind w into series of the eigenfunctions of the system (16):

$$\left.\begin{matrix} p \\ \rho \\ T \\ w \end{matrix}\right\} = \sum_{n,m,s,f} \begin{Bmatrix} (p_n^{m,f}(z)) \\ (\rho_n^{m,f}(z)) \\ (T_n^{m,f}(z)) \\ (w_n^{m,f}(z)) \end{Bmatrix} \Theta_n^{m,f}(\theta, z) \exp\left[j\left(m\lambda + 2\,\bar{f}\,\Omega t + s\,\Omega_a t\right)\right] \qquad (17)$$

with

$$\Theta_n^{m,f} = \sum_{n'} \xi_{n,n'}^{m,f}(z)\, P_n^m(c) \qquad (18)$$

Furthermore, we define

$$\mathrm{div}\cdot\vec{v}_{hor} = -\frac{2\,j\,f\,\Omega H_0}{p_0} \sum \frac{p_n^{m,f}}{h_n^{m,f}}\, \Theta_n^{m,f} \exp\left[j\left(m\lambda + 2\,\bar{f}\,\Omega t + s\,\Omega_a t\right)\right]. \qquad (19)$$

where we introduced the following abbreviations:

$f = (\omega + s\,\Omega_a)/2\Omega$

$c_0 = \sqrt{\gamma\, g H_0}$ is the velocity of sound

$\bar{f} = \omega/2\,\Omega$ the Coriolis parameter

ω is the angular frequency of the wave mode

Ω is the angular frequency of the sidereal day

Ω_a ($\ll \Omega$) is the angular frequency of the period of one year

$h_n^{m,f}$ is the equivalent depth of the mode of wave domain numbers (n,m,f)

H_0 is the scale height of the isothermal atmosphere

$\theta_n^{m,f}(\theta, z)$ is the eigenfunction determining the latitudinal structure of the wave mode.

For $m \geq 1$ and in the case that the dissipation terms can be neglected in (16), the eigenfunction $\theta_n^{m,f}$ degenerates to the well known Hough-function of tidal theory (Chapman and Lindzen, 1970). Contrary to this situation, our function $\theta_n^{m,f}$ depends not only on co-latitude θ but also on altitude z. Likewise, the equivalent depth $h_n^{m,f}$ which is real (either positive or negative) and which is constant within the lower atmosphere becomes height-dependent and generally complex within the dissipative atmosphere. This can readily be seen from equation (16) if we neglect the Coriolis force in the second and third equation of (16) and introduce the dissipation factors Z_{vis} from equation (3) and Z_{col} from equation (4). Then, the two equations of horizontal momentum in (16) become

$$u \sim -\frac{1}{2\,j\,f_u\,\Omega\,\rho_0\,r}\,\frac{\partial p}{\partial \theta}$$

$$v \sim -\frac{1}{2\,j\,f_v\,\Omega\,\rho_0\,s\,r}\,\frac{\partial p}{\partial \lambda} \tag{20}$$

with

$$f_u = f - j\,(0.6\,Z_{col} + (Z_{vis})_u)$$

$$f_v = f - j\,(Z_{col} + (Z_{vis})_v).$$

Taking for convenience

$$f_u \sim f_v \sim f_k \tag{21}$$

we obtain from equations (20) and (21)

$$\mathrm{div}\,\vec{v}_{hor} \sim -\frac{1}{2\,j\,f_k\,\Omega\,\rho_0\,r^2}\,\Delta'\,p \tag{22}$$

where Δ' is the horizontal component of the Laplace operator in spherical coordinates. The Laplace equation is satisfied by the spherical harmonic surface functions

$$Y_n^m = P_n^m\,(\theta)\,e^{jm\lambda} \tag{23}$$

and yields

$$\Delta'\,Y_n^m = -\,n\,(n+1)\,Y_n^m \tag{24}$$

Thus, the spherical functions become the eigenfunctions of system (16), and it follows from (18) and (19)

$$\theta_n^{m,f} \to P_n^m$$

Consequently,

$$\delta_{n,n'}^{m,f} \to \begin{cases} 1 & \text{for } n' = n \\ 0 & \text{for } n' \neq n \end{cases} \tag{25}$$

and

$$h_n^{m,f} \to \frac{4\, f_k\, f\, \Omega^2\, r^2}{n\,(n+1)\, g} \tag{26}$$

At a fixed height the eigenfunctions of (17) are a complete orthogonal system (Chapman and Lindzen, 1970). Therefore it is possible to develop any external heat source Q into a series of these eigenfunctions:

$$Q = \frac{\Omega\, p_0}{\kappa} \sum_{n\, m\, s\, f} J_n^{m,f}\, \theta_n^{m,f}\,(\theta,\, z)\, \exp\, [j\, (m\, \lambda + 2\, \bar{f}\, \Omega\, t + s\, \Omega_a\, t)] \tag{27}$$

with

$$\kappa = \frac{\gamma - 1}{\gamma}$$

$$J_n^{m,f} = \frac{\kappa\, Q_n^{m,f}}{\Omega\, p_0}$$

In (27) we allowed for any angular frequency ω of the energy source, not necessarily commensurable with the frequency of one sidereal day Ω. That

generalization may be important for the lunar tides and especially for the study of individual geomagnetic storm effects which contain a continual spectrum of periods (Volland and Mayr, 1971b). Moreover, it may be relevant for the study of tidal motions on the planets Venus and Mercury where solar day and sidereal day are quite different.

From equation (27) follows that

$$\frac{\partial}{\partial \lambda} = j\ m; \quad \frac{\partial}{\partial t} = j\ (\omega + s\ \Omega_a) \sim j\ \omega \tag{28}$$

where we generally neglect the time derivative of exp $[j\ s\ \Omega_a\ t]$.

Apparently, because of the height dependence of the eigenfunctions $\Theta_n^{m,f}$, these eigenfunctions are coupled with each other in a realistic atmosphere. In this approximate treatment we shall neglect that coupling in order to obtain tractable solutions. That is, we assume

$$\left.\begin{matrix} \left| \dfrac{1}{p_n^{m,f}} \dfrac{\partial p_n^{m,f}}{\partial z} \right| \\[2ex] \left| \dfrac{1}{w_n^{m,f}} \dfrac{\partial w_n^{m,f}}{\partial z} \right| \end{matrix}\right\} \gg \left| \frac{1}{\Theta_n^{m,f}} \frac{\partial \Theta_n^{m,f}}{\partial z} \right| \tag{29}$$

In paper I we shall show that this assumption is reasonable at least outside the height region between 100 and 200 Km.

We are now prepared to derive from system (16) the equations for the height dependence of the coefficients p, w, and T. Taking account of the various assumptions and approximations in sections 2 and 4, furthermore using the approximation equation (21) (which we shall abandon in the numerical study of paper I) we find from equation (16) the following system of ordinary differential equations of first order for ρ and w:

$$\frac{1}{j\,k_0}\,\frac{d\,\underline{e}}{d\,z} = \underline{K}\,\underline{e} + \underline{h} \tag{30}$$

with

$$\underline{e} = \begin{pmatrix} p_n^{m,f}/p_0 \\ \\ w_n^{m,f}/c_0 \end{pmatrix}$$

$$\underline{K} = \begin{pmatrix} -2\,j\,A\left(1 - \dfrac{f\,\kappa}{f_h}\right) & -2\,f\left(1 - \dfrac{H_0}{h_n^{m,f}} - \dfrac{f\,r}{f_h}\right) \\ \\ \dfrac{2\,A^2\,\kappa}{f_h} & -\dfrac{2\,j\,A\,f\,\kappa}{f_h} \end{pmatrix}$$

$$\underline{h} = -\frac{1}{f_h}\begin{pmatrix} j\,f \\ \\ A \end{pmatrix} J_n^{m,f}$$

$$k_0 = \frac{\Omega}{c_0}$$

$$A = \frac{1}{2\,k_0\,H_0} = \frac{\omega_a}{\Omega}$$

$$\omega_a = \frac{\gamma g}{2 c_0}$$

$$f_h = f - j Z_{hc} \quad (Z_{hc} \text{ from Equ. (2)})$$

while the temperature amplitude is

$$T_n^{m,f} = \frac{T_0}{2 f_h}\left(\frac{2 j A \kappa}{c_0} w_n^{m,f} + \frac{2 f \kappa}{p_0} p_n^{m,f} - j J_n^{m,f}\right) \tag{31}$$

Applying a well known matrix calculus (e.g. Volland, 1968), we find a phase integral solution of (30) (which is approximately valid as long as the elements of $\underline{K}$ in equation (30) change only slightly with height, and which is correct for constant elements of $\underline{K}$) to be

$$\underline{c} = j \int_{z_0}^{z} k_0 \exp\left[j \int_{\xi}^{z} k_0 \underline{N} \, d\xi \right] \underline{P}^{-1} \underline{h} \, d\xi + \exp\left[j \int_{z_0}^{z} k_0 \underline{N} \, d\xi \right] \underline{c}(z_0) \tag{32}$$

where

$$\underline{c} = \begin{pmatrix} a \\ \\ b \end{pmatrix} = \begin{pmatrix} a_0 \exp\left[\int (-j k_0 q + 1/2 H_0) \, d\xi\right] \\ \\ b_0 \exp\left[\int (j k_0 q + 1/2 H_0) \, d\xi\right] \end{pmatrix} \tag{33}$$

are the upgoing (a) and downgoing (b) characteristic waves which are connected with the physical wave parameters of equation (30) by the transformation

$$\underline{e} = \underline{P} \, \underline{c}. \tag{34}$$

The transformation matrix $\underline{P}$ is

$$\underline{P} = \begin{pmatrix} 1 & 1 \\ & \\ F_a & F_b \end{pmatrix} \tag{35}$$

with

$$\left.\begin{matrix} F_a \\ \\ F_b \end{matrix}\right\} = \frac{\pm q - j\,A\left(1 - \dfrac{2\,f\,\kappa}{f_h}\right)}{2\,f\left(1 - \dfrac{H_0}{h_n^{m,f}} - \dfrac{f\,\kappa}{f_h}\right)}$$

and $\underline{P}^{-1}$ its reciprocal.

Moreover, the eigenvalues q can be found from the normalization of the coefficient matrix $\underline{K}$:

$$\underline{N} = \underline{P}^{-1}\,\underline{K}\,\underline{P} = \begin{pmatrix} -q - j\,A & 0 \\ & \\ 0 & q - j\,A \end{pmatrix}^{*} \tag{36}$$

as

$$q_n^{m,f} = -A\,(\alpha_n^{m,f} + j\,\beta_n^{m,f}) = -A\sqrt{-1 + \frac{4\,H_0\,\kappa\,f}{h_n^{m,f}\,f_h}} \tag{37}$$

The up- and downgoing waves are so defined that their amplitudes remain bounded in the direction of propagation. Thus, the imaginary part of the eigenvalue of the upgoing waves is negative, and positive for the downgoing waves. Moreover, we know from the theory of gravity waves (Hines, 1960) that the

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* Obviously, the subscripts and superscripts $\binom{m,f}{n}$ should be added to the matrices $\underline{e}$, $\underline{c}$, $\underline{h}$, $\underline{P}$, $\underline{K}$, $\underline{N}$ as well as to the parameters q, F_a, and F_b. We omit these indices for convenience as long as no confusion is to be expected.

vertical phase velocity of upgoing gravity waves is negative. Thus, the real part of the eigenvalue of upgoing waves is negative, too, and the terms α and β in equation (37) are both positive definite. That definition includes the two special cases of wave propagation within the nondissipative atmosphere in which $f_k = f_h = f$. There, it is $\beta = 0$ for the propagation modes, and $\alpha = 0$ for the evanescent or trapped modes. Trapped modes with negative equivalent depths have an imaginary eigenvalue of $\beta > 1$. These waves can not transport wave energy in the vertical direction. Their wave energy leaks away vertically to such extent that the wave amplitude of the upgoing wave decreases according to

$$a \propto \exp \{- k_0 (\beta - 1) A (z - z_0)\} \tag{38}$$

Thus, the adiabatic increase of the wave amplitude due to the exponential factor $1/2\ H_0$ in equation (33) is overcompensated by the leakage effect. We shall see in the following that generally within the dissipative thermosphere all wave modes have imaginary terms $\beta > 1$, but real terms α different from zero. Therefore, we shall call waves of that type quasi-evanescent modes. They fulfill the radiation condition as well as the condition that the kinetic energy density is bounded at $z \to \infty$. Our eigenvalue equation (37) of course degenerates as it should to the eigenvalue of the nondissipative atmosphere for $f_h = f_k = f$ (Chapman and Lindzen, 1970).

5. SPECIAL SOLUTIONS OF THE GENERATION AND PROPAGATION OF ATMOSPHERIC WAVES

We now want to discuss two special solutions of equation (30) which have some relevance to the generation and propagation of waves within the atmosphere. We shall restrict ourselves in this section to an isothermal atmosphere with constant dissipation factors Z; that is, constant elements of the coefficient matrix $\underline{K}$ in equation (30). A more quantitative treatment including realistic temperature profiles and vertical profiles of the dissipation factors will be given in paper I.

a. Infinitely extended isothermal atmosphere

We first discuss the simplest case of wave propagation within an isothermal atmosphere infinitely extended in the vertical direction which shall simulate wave generation within the thermosphere. We assume an external heat input with the vertical profile (see equation (5))

$$J_n^{m,f} = J^0(z_0)\, e^{(1-\epsilon)(z-z_0)/2H_0} \quad \text{for } z_0 \leq z \leq z_1 \tag{39}$$

Outside that region the heat input is assumed zero. For the boundary conditions we adopt the simple radiation conditions namely that waves generated within the region $z_0 \leq z \leq z_1$ by the heat source of equation (30) can only leave that region through z_0 and z_1, respectively. This implies that

$$a(z_0) = b(z_0) = 0$$

The explicit solution of (30) then becomes

$$\left.\begin{aligned} a(z) &= 0 \\ b(z) &= b(z_0) \exp\left[j\, k_0 (q - j\, A)(z - z_0)\right] \end{aligned}\right\} \quad \text{for } z \leq z_0 \tag{41a}$$

$$\left.\begin{aligned} a(z) &= G_a \{1 - \exp\left[- j\, k_0 (q + j\, \epsilon\, A)(z - z_0)\right]\} \\ b(z) &= G_b \{1 - \exp\left[j\, k\, (q - j\, \epsilon\, A)(z - z_1)\right]\} \end{aligned}\right\} \quad \text{for } z_0 \leq z \leq z_1 \tag{41b}$$

$$\left.\begin{aligned} a(z) &= a(z_1) \exp\left[- j\, k_0 (q + j\, A)(z - z_1)\right] \\ b(z) &= 0 \end{aligned}\right\} \quad \text{for } z_1 \leq z \tag{41c}$$

with

$$G_a = - \frac{j\, J\, (f\, F_b + j\, A)}{f_h\, (F_b - F_a)(q + j\, \epsilon\, A)}$$

$$G_b = - \frac{j\, J\, (f\, F_a + j\, A)}{f_h\, (F_b - F_a)(q - j\, \epsilon\, A)}$$

The physical wave parameters are according to Equations (31) and (34)

$$\begin{aligned} \frac{w_n^{m,f}}{c_0} &= a + b \\ \frac{p_n^{m,f}}{p_0} &= F_a\, a + F_b\, b \\ \frac{T_n^{m,f}}{T_0} &= \frac{1}{2\, f_h} \left[2\, \kappa\, (f\, F_a + j\, A)\, a + 2\, \kappa\, (f\, F_b + j\, A)\, b - j\, J\right] \end{aligned} \tag{42}$$

where

$$J = J_n^{m,f} = \frac{\kappa\, Q_n^{m,f}}{\Omega\, p_0}$$

from equations (39) and (27).

With the boundary conditions (40) we implicitly neglected the reflection of waves (generated within $z_0 < z < z_1$) at the earth's surface. In an exact treatment one would have to add to the upgoing wave in (41) the wave

$$\tilde{a}(z) = -b(z_0)\exp\{-2jk_0 q z_0 - jk_0(q + jA)(z - z_0)\} \qquad (43)$$

which is the downgoing wave of equation (41a) reflected at the earth's surface. For the trapped modes this reflected wave is insignificant at $z_0 \sim 100$ km. For the propagation waves, $\tilde{a}$ is only significant in the immediate vicinity of the lower boundary z_0 and it is entirely negligible when compared with the wave generated below z_0 (see section 5b). Thus, our solution (41) describes sufficiently well the generation and propagation of waves within the thermosphere.

From the approximated equivalent depth at thermospheric altitudes in equation (26) and from the equation of the eigenvalue q in (37) we estimate that a wave mode becomes quasi-evanescent in a region where

$$|Z_{hc}\, Z_{kin}| \gtrsim f^2$$

which is certainly valid above about 200 km altitude. There the imaginary part of the eigenvalue in (37) becomes $\beta > 1$, and we find from equation (41b) that the waves approach the asymptotic values

$$\left.\begin{array}{l} a \to G_a \\ \\ b \to G_b \end{array}\right\} \quad \text{for } z_1 \to \infty \tag{44}$$

if the heat source equation (39) is extended into infinite. Then, the wave amplitudes of equation (42) approach the asymptotic values

$$\frac{w_n^{m,f}}{c_0} \to G_a + G_b = \frac{f\, J\, (2\, H_0/h - 1 - \epsilon)}{A\, [(\epsilon^2 - 1)\, f_h + 4\, \kappa\, f\, H_0/h]} \sim \frac{J\, [n\,(n+1) - 2\,(1+\epsilon)\, f_k\, f\, \xi^2\, \gamma]}{2\, A\, [(\epsilon^2 - 1)\, f_k\, f_h\, \xi^2\, \gamma + \kappa\, n\,(n+1)]}$$

$$\frac{p_n^{m,f}}{p_0} \to F_a\, G_a + F_b\, G_b = \frac{j\, J\, (1+\epsilon)}{(\epsilon^2 - 1)\, f_h + 4\, \kappa\, f\, H_0/h} \sim \frac{j\, J\, (1+\epsilon)\, f_k\, \xi^2\, \gamma}{[(\epsilon^2 - 1)\, f_k\, f_h\, \xi^2\, \gamma + \kappa\, n\,(n-1)]} \tag{45}$$

$$\frac{T_n^{m,f}}{T_0} \to \frac{(1-\epsilon)}{2}\, \frac{p_n^{m,f}}{p_0}$$

and from the last Equation (16),

$$\frac{\rho_n^{m,f}}{\rho_0} \to \frac{(1+\epsilon)}{2}\, \frac{p_n^{m,f}}{p_0}$$

with

$$\xi = \frac{\Omega}{c_0}\, r$$

Here, the last equations on the right hand side of equations (45) have been calculated with help of the approximate equivalent depth of equation (26).

From equations (20) and (26) we find the assymptotic wave amplitudes of the horizontal winds

$$\left[\frac{u}{c_0}\right]_n^{m,f} \rightarrow - C_n \frac{d P_n^m}{d\theta} \exp\{j(m\lambda + 2 \bar{f} \Omega t + s \Omega_a t)\}$$

$$\left[\frac{v}{c_0}\right]_n^{m,f} \rightarrow - j\, m\, C_n \frac{P_n^m}{\sin\theta} \exp\{j(m\lambda + 2 \bar{f} \Omega t + s \Omega_a t)\} \tag{46}$$

with

$$C_n := \frac{J (1 + \epsilon) \xi}{2 [(\epsilon^2 - 1) f_k f_h \xi^2 \gamma + \kappa n (n + 1)]}$$

Since f_h and f_k tend to become constant with altitude (see section 2) the relative amplitudes of the waves also approach constant values if $\epsilon = 1$, and we obtain for $z \rightarrow \infty$

$$p/p_0 = \rho/\rho_0 \rightarrow \frac{2 j J f_k \xi^2 \gamma}{\kappa n (n + 1)}$$

$$u/c_0 \propto v/c_0 \propto \frac{1}{n (n + 1)}$$

$$T/T_0 \rightarrow 0 \tag{47}$$

$$w/w_0 \rightarrow \frac{1}{2 A \kappa} (1 - 4 f_k f \xi^2 \gamma \kappa / n (n + 1))$$

That means, the thermosphere behaves like a low pass filter which suppresses the pressure, density and horizontal wind amplitudes of the wave modes proportional to $1/n(n+1)$. The same is the case for the vertical wind for low wave domain numbers n, while the vertical winds tends to become independent of n for large numbers n. The temperature amplitude approaches zero at great heights which is not consistent with the observations. However, it will be seen in the following parts of this paper that this decrease with altitude occurs very slowly and that the assymptotic value is only reached at heights of several thousands of kilometers. Below 400 km, the temperature remains nearly constant. As already mentioned before the temperature amplitude is significantly affected by the neglect of heat conduction waves. In paper I it will be shown that due to heat conduction waves the temperature amplitude slightly increases toward a finite assymptotic value at high altitudes.

We can simulate this influence of heat conduction waves in our simplified model by introducing in equation (5) a value of

$$\epsilon \simeq 0.75 \tag{48}$$

which corresponds to a heat input that decreases with altitude somewhat slower than the mean pressure. Then we find the ratio between temperature and density from (45)

$$\frac{T/T_0}{\rho/\rho_0} = \frac{1-\epsilon}{1+\epsilon} \sim 0.14$$

which is slightly smaller than in the Jacchia-model (Jacchia, 1964).

Taking the value of ϵ from equation (48) and assuming plausible physical parameters for the thermosphere we find from equation (45) a formula for the effective assymptotic magnitudes of pressure, density, temperature and horizontal winds

$$\left.\begin{array}{l} p/p_0 \\ \rho/\rho_0 \\ T/T_0 \\ u/c_0 \\ v/c_0 \end{array}\right\} \propto \frac{Q_n^{m,f}}{0.4 + n(n+1)} \tag{49}$$

This formula is valid within the height range between about 350 and 400 km and is a very convenient half empirical formula (Volland and Mayr, 1971b) for tidal and planetary waves at thermospheric heights as varified by the theory (see paper I and the results of the following parts of this paper).

b. Wave generation within the lower atmosphere

We now want to consider wave generation within the lower atmosphere which is important for the study of the tidal propagation modes. The relative amplitudes of tidal propagation modes with zero attenuation ($\beta = 0$) increase with height according to exp (Az) within the nondissipative atmosphere. We therefore expect

significant wave amplitudes for the upgoing waves of those modes at the base of the thermosphere which we have to add to the upgoing waves generated within the thermosphere. The lower boundary condition at the earth's surface is that vertical winds should disappear. Then, taking a heat source of the form of equation (39) within the height range between $0 \leqq z \leqq z_0$ we obtain from equation (32)

$$a(z) = G_a \{1 - \exp[-j k_0 (q - j \epsilon A) z]\} - G_b \{1 - \exp[-j k_0 (q - j \epsilon A) z_0]\} \exp[-j k_0 (q + j A) z]$$

$$\text{for } 0 \leqq z \leqq z_0 \qquad \text{(50a)}$$

$$b(z) = G_b \{1 - \exp[j k_0 (q - j \epsilon A)(z - z_0)]\}$$

and

$$a(z) = a(z_0) \exp[-j k_0 (q + j A)(z - z_0)]$$

$$\text{for } z_0 \leqq z \qquad \text{(50b)}$$

$$b(z) = 0$$

Because of $\beta \geqq 0$, it is at the earth's surface ($z = 0$)

$$a = -b$$

$$\text{for } z = 0 \qquad \text{(51)}$$

$$b \sim G_b$$

Thus

$$\frac{w_n^{m,f}}{c_0} = 0$$

$$\text{for } z = 0; \; f_h = f$$

$$\frac{p_n^{m,f}}{p_0} \sim - \frac{j(f F_a + j A)}{f(q - j \epsilon A)}$$

At the base of the thermosphere ($z = z_0$) it is for propagation modes:

$$a(z_0) \sim -\{G_a \exp[-j k_0 (q + j \epsilon A) z_0] + G_b \exp[-j k_0 (q + j A) z_0]\} \quad \text{for } \beta < 1, \tag{52a}$$

$$b(z_0) = 0$$

and for evanescent modes:

$$\left.\begin{array}{l} a(z_0) \sim G_a \\ b(z_0) = 0 \end{array}\right\} \text{for } \beta > 1. \tag{52b}$$

The observed atmospheric wave modes are the sum of the wave modes generated by the energy input within the lower atmosphere (equation (50)) and the energy input within the thermosphere (equation (41)).

6. CONCLUSION

In this paper we developed a three-dimensional model of thermospheric dynamics in terms of the eigenfunctions of the atmospheric system. These eigenfunctions or wave modes are excited by solar heating from XUV-radiation from particle precipitation and joule heating in the auroral zone during geomagnetic disturbances and from energy dissipation of waves from the lower atmosphere. We determined formulae for both solar heat sources in terms of the eigenfunctions of the atmospheric system. That series contains tidal components depending on local time and planetary components depending on seasonal time. We estimated the

relative importance of auroral heating when compared with solar XUV-heating and found that the average auroral heating contributes not more than about 15% to the total solar heat input.

Approximate analytic solutions for the generation and propagation of atmospheric waves within the dissipative thermosphere excited by the solar heat input have been derived. It was shown that the eigenfunctions which are the Hough-functions within the non-dissipative lower atmosphere, change into the spherical surface functions within the dissipative thermosphere. The amplitudes of density and horizontal winds for the various wave modes are shown to decrease proportional to $1/n^2$ where n is the zonal wave domain number of the spherical harmonics. Therefore, only wave modes with low wave domain number n are significant at thermospheric heights.

In two further parts of this paper we shall discuss in detail the characteristics of various tidal and planetary wave modes, especially the height dependence of the eigenfunctions and equivalent depths and the change in the latitudinal structure of the wave modes. In an additional paper (Volland and Mayr, 1971a) numerical full wave calculations are carried out to develop a more sophisticated and quantitative thermosphere model.

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